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A GENERAL FIXED POINT THEOREM IN ORBITALLY 0 - COMPLETE PARTIAL METRIC SPACES

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Abstract. In this paper a general fixed point theorem for a pair of mappings in orbitally 0 - complete partial metric space is proved, generalizing Theorem 2 [13].

Keywords: *fixed point, orbitally 0 - complete partial metric space, implicit relation.*

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1. Introduction

In 1974, Ćirić [6] has first introduced orbitally complete metric spaces and orbitally continuous function. Let f be a self mapping of a metric spaces (X,d). If for $x_0 \in X$, every Cauchy sequence of the orbit $O_{x_0}(f)=\{x_0, fx_0, f^2x_0, ...\}$ is convergent to a point $y \in X$ then X is said to be orbitally complete in x_0. If f is orbitally complete at each $x_0 \in X$, then f is said to be orbitally complete. Every complete metric space is f orbitally complete for every function f. An orbitally complete metric space may not be a complete metric space ([20], Examples 4, 5).

A function $f:(X,d)\to(X,d)$ is said to be orbitally continuous at a point $x \in X$ if fy_n converges to fz for every subsequence $y_n \in O_x(f)$ which converges to a point $z \in X$.

The function f is said to be orbitally continuous if it is orbitally continuous at each $x \in X$.

Example 1. Let $X=[0,\infty)$ be and $d(x,y)=|x-y|$. The mapping $f:(X,d)\to(X,d)$, $fx=\frac{x}{x+1}$ is orbitally continuous.

An orbitally continuous mapping may not be continuous ([20], Examples 4, 5).

Some fixed point results for mappings in orbitally complete metric spaces are obtained in [2], [7], [15], [16] and in other papers.

In 1994, Matthews [12] introduced the concept of partial metric space as a part of the study of denotional semantics of dataflow networks and proved the Banach contraction principle in such spaces.

Quite recently, in [1], [4], [5], [8], [10] and in other papers, some fixed point theorems under various contractive conditions in complete partial metric spaces are proved.

Recently, in [10] the authors initiated the study of fixed points in orbitally complete partial metric spaces. In [9], [13], [14] new results are obtained.

Romaguera [19] introduced the notion of 0 - Cauchy sequence, 0 - complete partial metric space and proved some characterizations of partial metric spaces in terms of completeness and 0 - completeness.

Some fixed point theorems for mappings in 0 - complete partial metric spaces are proved in [3], [11] and in other papers.

Quite recently, Nashine et al. [14] proved some fixed point theorems for mappings in orbital 0 - complete partial metric spaces.

Several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [17], [18] and in other papers.

The method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, b - metric spaces, Hilbert spaces, ultra - metric spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two and three metric spaces, for single valued mappings, hybrid pairs of mappings and multi - valued mappings.

Recently, the method is used in the study of fixed points for mappings satisfying a contractive / extensive condition of integral type, in fuzzy metric spaces, probabilistic metric spaces, intuitionistic metric spaces, G - metric spaces and G_p - metric spaces. With this method, the proofs of some fixed point theorems are more simple. Also, the method allows the study of local and global properties of fixed point structures.

The study of fixed point for mappings satisfying an implicit relation in partial metric spaces is initiated in [21].

2. Preliminaries

Definition 1 ([12]). Let X be a nonempty set. A function $p : X \times X \to \mathbb{R}_+$ is said to be a partial metric on X if for any $x, y, z \in X$, the following conditions hold:

$(P_1) : p(x,x) = p(y,y) = p(x,y)$ if and only if $x = y$,

$(P_2) : p(x,x) \le p(x,y),$

$(P_3) : p(x,y) = p(y,x),$

$(P_4) : p(x,z) \le p(x,y) + p(y,z) - p(y,y).$

The pair (X, p) is called a partial metric space.

If $p(x,y) = 0$ then by (P_1) and (P_2), $x = y$, but the converse is not always true.

Each partial metric p on X generates a T_0 - topology τ_p on X which has as base the family of open p - balls $\{B_p(x,\varepsilon): x \in X,\ \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) \le p(x,x) + \varepsilon$ for all $x \in X$ and $\varepsilon > 0\}$.

A sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x \in X$, $x_n \to x$, with respect to τ_p if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

If p is a partial metric on X, then the function

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$$

defines a metric on X.

Furthermore, a sequence $\{x_n\}$ converges in (X,d_p) if and only if

$$\lim_{n,m\to\infty} p(x_n,x_m) = \lim_{n\to\infty} p(x_n,x) = p(x,x). \tag{1}$$

Lemma 1 ([1], [10]). Let $\{x_n\} \to z$ as $n \to \infty$ in a partial metric space (X,p), where $p(z,z) = 0$. Then, $\lim_{n\to\infty} p(x_n,y) = p(z,y)$ for any $y \in X$.

Definition 2. Let (X,p) be a partial metric space.

a) A sequence $\{x_n\}$ in (X,p) is called Cauchy ([12]) if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite.

The space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges in X.

b) A sequence $\{x_n\}$ in (X,p) is called 0 - Cauchy ([19]) if $\lim_{n,m\to\infty} p(x_n,x_m) = 0$.

The space (X,p) is said to be 0 - complete if every 0 - Cauchy sequence in X converges with respect to τ_p to a point x such that $p(x,x) = 0$.

Remark 1. If (X,p) is complete, then it is 0 - complete. The converse is not true ([19]).

Definition 3 ([14]). Let (X,p) be a partial metric space and $T: X \to X$ a mapping. A partial metric space (X,p) is said to be T - orbitally complete if every 0 - Cauchy sequence contained in $O_x(T)$, for some $x \in X$, converges in X to a point z such that $p(z,z) = 0$.

Remark 2. Every 0 - complete partial metric space is T - orbitally complete for any T, but the converse does not hold.

Definition 4 ([14]). A self mapping T defined on a partial metric space (X,p) is said to be orbitally continuous at a point z in X if for any sequence $\{x_n\} \in O_x(T)$, for some $x \in X$, $x_n \to z$ as $n \to \infty$ (in τ_p) implies $Tx_n \to Tz$ as $n \to \infty$ in τ_p.

Clearly, every continuous self mapping of a partial metric space is orbitally continuous but not conversely ([14]).

Definition 5 ([14]). Let S,T be two self mappings on a partial metric space (X,p).

1) If for a point $x \in X$, a sequence $\{x_n\}$ in X such that

$$x_{2n+1} = Sx_{2n}, x_{2n+2} = Tx_{2n+1}, n = 0,1,2,...,$$

then the set $O_{x_0}(S,T) = \{x_n : n = 1,2,...\}$ is called the orbit of (S,T).

2) The space (X,p) is said to be (S,T) - orbitally 0 - complete at x_0 if every 0 - Cauchy sequence in $O_{x_0}(S,T)$ converges to a point $z \in X$ such that $p(z,z) = 0$

3) The maps S,T are said to be orbitally continuous at x_0 if they are continuous on $O_{x_0}(S,T)$.

The following result is obtained in [14].

Theorem 1 (Theorem 2 [14]). Let (X,p) be a partial metric space, $S,T : X \to X$ be two given mappings such that

$$p(Tx,Sy) \le \frac{ap(x,Tx)\,p(y,Ty)}{1+p(x,y)} + bp(x,y) +$$
$$+c[p(x,Tx)+p(y,Ty)] + d[p(x,Sy)+p(y,Tx)],$$

for all $x,y \in O_{x_0}(S,T)$, where $a,b,c,d \ge 0$ with $a+b+2c+2d < 1$.

We assume that (X,p) is (S,T) - orbitally 0 - complete at x_0. Then S and T have a common fixed point $z \in X$ such that $p(z,z) = p(z,Tz) = p(z,Sz) = 0$.

If, moreover, each common fixed point z of S and T in $\overline{O_{x_0}(S,T)}$ satisfies $p(z,z) = 0$, then the common fixed point of T and S in $\overline{O_{x_0}(S,T)}$ is unique.

The purpose of this paper is to prove a fixed point theorem for a pair of maps (S,T) satisfying an implicit relation which generalizes Theorem 2.1.

3. Implicit relations

Definition 6. Let $\mathcal{F}_{p0}$ be the set of all continuous functions $F(t_1,...,t_5) : \mathbb{R}_+^5 \to \mathbb{R}$ satisfying:

(F_1): F is nonincreasing in t_5,

(F_2): There exists $h \in [0,1)$ such that for all $u,v \ge 0$,

(F_{2a}): $F(u,v,v,u,u+v) \le 0$ implies $u \le hv$, and

(F_{2b}): $F(u,v,u,v,u+v) \le 0$ implies $u \le hv$,

(F_3): $F(t,t,0,0,2t) > 0, \forall t > 0$.

In the following examples, property (F_1) is obviously.

Example 2. $F(t_1,...,t_5)=t_1-\frac{at_2t_3}{1+t_2}-bt_2-c(t_3+t_4)-dt_5$, where $a,b,c,d\geq 0$ and $a+b+2c+2d<1$.

(F_2): Let $u,v\geq 0$ be and

$(F_{2a}):F(t,v,v,u,u+v)=u-\frac{auv}{1+v}-bv-c(u+v)-d(u+v)\leq 0$. If $v>0$, then $u-au-bv-c(u+v)-d(u+v)\leq 0$. If $u>v$, then $u[1-(a+b+2c+2d)]\leq 0$, a contradiction. Hence, $u\leq v$ which implies $u\leq hv$, where $0<h=a+b+2c+2d<1$. If $v=0$, then $u=0$ and $u\leq hv$.

Similarly, $F(u,v,u,v,u+v)\leq 0$ implies $u\leq hv$.

$(F_3):F(t,t,0,0,2t)=t[1-(b+2d)]>0,\forall t>0$.

Example 3. $F(t_1,...,t_5)=t_1-k\max\left\{t_2,t_3,t_4,\frac{t_5}{2}\right\}$, where $k\in[0,1)$.

(F_2): Let $u,v\geq 0$ be and $(F_{2a}):F(t,v,v,u,u+v)=u-k\max\left\{u,v,\frac{u+v}{2}\right\}\leq 0$. If $u>v$, then $u(1-k)\leq 0$, a contradiction. Hence, $u\leq v$ which implies $u\leq hv$, where $0<h=k<1$.

Similarly, $F(u,v,u,v,u+v)\leq 0$ implies $u\leq hv$, where $0<h=k<1$.

$(F_3):F(t,t,0,0,2t)=t(1-k)>0,\forall t>0$.

Example 4. $F(t_1,...,t_5)=t_1-k\max\{t_2,t_3,t_4,t_5\}$, where $k\in\left[0,\frac{1}{2}\right)$.

Example 5. $F(t_1,...,t_5)=t_1-a\max\{t_3,t_4\}-bt_5$, where $a,b\geq 0$ and $a+2b<1$.

Example 6. $F(t_1,...,t_5)=t_1-at_2t_3-bt_3t_4-ct_5^2$, where $a,b,c\geq 0$ and $a+b+4c<1$

Example 7. $F(t_1,...,t_5)=t_1^2+\frac{t_1}{1+t_5}-(at_2^2+bt_3^2+ct_4^2)$, where $a,b,c\geq 0$ and $a+b+c<1$.

Example 8. $F(t_1,...,t_5)=t_1-at_2-bt_3-c\max\{2t_4,t_5\}$, where $a,b,c\geq 0$ and $a+b+2c<1$.

Example 9. $F(t_1,...,t_5)=t_1^2-t_1(at_2+bt_3+ct_4)-dt_5^2$, where $a,b,c,d\geq 0$ and $a+b+c+4d<1$.

4. Main results

Theorem 2. Let (X,p) be a partial metric space and $S,T:X\to X$ be two mappings satisfying inequality

$$F(p(Tx,Sy),p(x,y),p(x,Tx),p(y,Sy),p(x,Sy)+p(y,Tx))\leq 0, \qquad (1)$$

for all $x, y \in \overline{O_{x_0}(S,T)}$ for some $x_0 \in X$ and $F \in \mathsf{F}_{p0}$. If (X, p) is (S,T) - orbitally 0 - complete at x_0, then T and S have a common fixed point $z \in X$ such that $p(z,z) = p(z,Tz) = p(z,Sz) = 0$. If moreover, each common fixed point z of S and T in $\overline{O_{x_0}(S,T)}$ satisfies $p(z,z) = 0$, then the common fixed point of S and T in $O_{x_0}(S,T)$ is unique.

Proof. First we prove that if $z = Sz$ and $p(z,z) = 0$, then z is a common fixed point of T and S.

By (1) we obtain

$$F(p(Tz,Sz), p(z,z), p(z,Tz), p(z,Sz), p(z,Sz) + p(z,Tz)) \leq 0,$$
$$F(p(Tz,z), 0, p(z,Tz), 0, p(z,Tz) + 0) \leq 0.$$

By (F_{2a}) we have $p(z,Tz) = 0$, i.e. $z = Tz$ and z is a common fixed point of S and T.

Similarly, if $Tz = z$ and $p(z,z) = 0$, by (F_{2a}) we obtain $p(z,Sz) = 0$, i.e. z is a common fixed point of T and S.

We define a sequence $\{x_n\}$ in X as follows

$$x_{2n+1} = Sx_{2n} \text{ and } x_{2n+2} = Tx_{2n+1}, \text{ for } n = 0,1,2,... \tag{2}$$

If there exists $n_0 \in \mathrm{N}$ such that $p\left(x_{n_0}, Sx_{n_0}\right) = 0$ or $p\left(x_{n_0}, Tx_{n_0}\right) = 0$ for some n then T and S have a common fixed point. We suppose that $p(x_n, x_{n+1}) > 0$, for $n = 0,1,2,...$.

By (1) and (2), for $x = x_{2n+1}$ and $y = x_{2n}$, we obtain

$$\begin{aligned} &F(p(Tx_{2n+1}, Sx_{2n}), p(x_{2n+1}, x_{2n}), p(x_{2n+1}, Tx_{2n+1}), \\ &p(x_{2n}, Sx_{2n}), p(x_{2n+1}, Sx_{2n}) + p(x_{2n}, Tx_{2n+1})) \leq 0, \\ &F(p(x_{2n+2}, x_{2n+1}), p(x_{2n+1}, x_{2n}), p(x_{2n+1}, x_{2n+2}), \\ &p(x_{2n}, x_{2n+1}), p(x_{2n+1}, x_{2n+1}) + p(x_{2n}, x_{2n+2})) \leq 0. \end{aligned} \tag{3}$$

By (P_4),

$$p(x_{2n}, x_{2n+1}) \leq p(x_{2n}, x_{2n+1}) + p(x_{2n+1}, x_{2n+2}) - p(x_{2n+1}, x_{2n+1}).$$

By (F_1) and (3) we obtain

$$\begin{aligned} &F(p(x_{2n+2}, x_{2n+1}), p(x_{2n+1}, x_{2n}), p(x_{2n+1}, x_{2n+2}), \\ &p(x_{2n}, x_{2n+1}), p(x_{2n}, x_{2n+1}) + p(x_{2n+1}, x_{2n+2})) \leq 0. \end{aligned}$$

By (F_{2b}) we obtain

$$p(x_{2n+2}, x_{2n+1}) \leq hp(x_{2n+1}, x_{2n}), \text{where} h = \max\{h_1, h_2\}.$$

By (1) and (2), for $x = x_{2n-1}$ and $y = x_{2n}$, we obtain

$$F(p(Tx_{2n-1},Sx_{2n}),p(x_{2n-1},x_{2n}),p(x_{2n-1},Tx_{2n-1}),$$
$$p(x_{2n},Sx_{2n}),p(x_{2n-1},Sx_{2n})+p(x_{2n},Tx_{2n-1}))\le 0,$$
$$F(p(x_{2n},x_{2n+1}),p(x_{2n-1},x_{2n}),p(x_{2n-1},x_{2n}),$$
$$p(x_{2n},x_{2n+1}),p(x_{2n-1},x_{2n+1})+p(x_{2n},x_{2n-1}))\le 0. \tag{4}$$

By (P_4),

$$p(x_{2n-1},x_{2n+1})\le p(x_{2n-1},x_{2n})+p(x_{2n},x_{2n+1})-p(x_{2n},x_{2n}).$$

By (4) and (F_1) we obtain

$$F(p(x_{2n},x_{2n+1}),p(x_{2n-1},x_{2n}),p(x_{2n-1},x_{2n}),$$
$$p(x_{2n},x_{2n+1}),p(x_{2n-1},x_{2n})+p(x_{2n},x_{2n+1}))\le 0.$$

By (F_{2a}) we obtain

$$p(x_{2n},x_{2n+1})\le hp(x_{2n-1},x_{2n}).$$

Hence

$$p(x_n,x_{n+1})\le hp(x_{n-1},x_n)\le \ldots \le h^n p(x_0,x_1). \tag{5}$$

Then for each $m>n$, $n,m\in \mathrm{N}$ by (5) and (P_4) we have

$$\begin{aligned} 0 \quad \le \quad & p(x_n,x_{n+m})\le p(x_n,x_{n+1})+p(x_{n+1},x_{n+2})+\ldots+p(x_{m-1},x_m) \\ \le \quad & h^n\left(1+h+\ldots+h^{m-1}\right)p(x_0,x_1) \\ \le \quad & \frac{h^n}{1-h}p(x_0,x_1). \end{aligned}$$

Thus, $\lim_{n,m\to\infty}p(x_n,x_m)=0$. This implies that $\{x_n\}$ is a 0 - Cauchy sequence in the partial metric space $O_{x_0}(S,T)$. Since X is (S,T) - orbitally 0 - complete at x_0, then there exists a $z\in X$ with $\lim_{n\to\infty}x_n=z$ and $p(z,z)=0$.

We prove that z is a fixed point for S.

By (1), for $x=x_{2n+1}$ and $y=z$, we obtain

$$F(p(Tx_{2n+1},Sz),p(x_{2n+1},z),p(x_{2n+1},Tx_{2n+1}),$$
$$p(z,Sz),p(x_{2n+1},Sz)+p(z,Tx_{2n+1}))\le 0,$$
$$F(p(x_{2n+2},Sz),p(x_{2n+1},z),p(x_{2n+1},x_{2n+2}),$$
$$p(z,Sz),p(x_{2n+1},Sz)+p(z,x_{2n+2}))\le 0.$$

Letting n tend to infinity, by Lemma 1 and (5) we obtain

$$F(p(z,Sz),0,0,p(z,z),p(z,Sz)+0)\le 0.$$

By (F_{2a}) we obtain $p(z,Sz)=0$. Hence $z=Sz$ and z is a fixed point of S. By the first part of the proof, z is a common fixed point.

Suppose now that each common fixed point z of T and S in $\overline{O_{x_0}(S,T)}$ satisfies $p(z,z)=0$. We claim that there is a unique common fixed point for T

and S in $\overline{O_{x_0}(S,T)}$. Assume the contrary, i.e. $p(u,Su)=p(u,Tu)=0$ and $p(v,Sv)=p(v,Tv)=0$ but $u\neq v$. Then, by (1), for $x=u$ and $y=v$, we obtain

$$F(p(Tu,Sv), p(u,v), p(u,Tu),$$
$$p(v,Sv), p(u,Sv)+p(v,Tu))\leq 0,$$
$$F(p(u,v), p(u,v), 0, 0, 2p(u,v))\leq 0,$$

a contradiction of (F_3). Hence, $u=v$.

Theorem 3. Let (X,p) be a partial metric space such that X is T - orbitally 0 - complete at some $x_0\in X$ and

$$F(p(Tx,Ty), p(x,y), p(x,Tx), p(y,Ty), p(x,Ty)+p(y,Tx))\leq 0, \tag{6}$$

for all $x,y\in\overline{O_{x_0}(T)}$ and F satisfies properties $(F_1),(F_{2a}),(F_3)$. Then T has a fixed point. If moreover, each fixed point $z\in X$ in $\overline{O_{x_0}(T)}$ satisfies $p(z,z)=0$, then the fixed point is unique.

Remark 3.

1) Theorems 2 and 3 improve some results from the literature to a more general case. Indeed, by Theorem 2 and Example 2 we obtain Theorem 1, and by Theorem 3 and Example 2 we obtain Corollary 6 [14].

3) By Theorems 2 and 3 and Examples 3 - 9 we obtain new particular results.

References

1. Abdeljawad T., Karapinar E., Tas K., (2011) Existence and uniqueness of a common fixed point on a partial metric space, *Appl. Math. Lett.*, 24(11), 1900-1904.
2. Achari J., (1976) Ćirić's non - unique fixed point, *Mat. Vesnik*, 13, 255-257.
3. Ahmad A.E.B., Fadail Z.M., Rajićo V.C., Radenović S., (2012) Nonlinear contractions in 0 - complete partial metric spaces, *Abstr. Appl. Anal.*, Article ID 451239.
4. Altun I., Sola F., Simse H.K., (2010) Generalized contractions on partial metric spaces, *Topology Appl.*, 157(18), 2778-2785.
5. Chi R.P., Karapinar E., Thanh T.D., (2012) A generalized contraction principle in partial metric spaces, *Math. Comput. Modelling*, 53, 1673-1681.
6. Ćirić L., (1974) On some maps with a nonunique fixed point, *Publ. Inst. Math. New Ser.*, 17(3), 52-58.
7. Ćirić L., Jotić N., (1998) A further extension of maps with nonunique fixed points, *Mat. Vesnik*, 50(1–2), 1 - 4.
8. Kadelburg Z., Nashine H. K., Radenović S., (2013) Fixed point results under various contractive conditions in partial metric spaces, *RACSAM*, 10, 241-256.
9. Karapinar E., (2012) Ćirić's types nonunique fixed point theorems on partial metric spaces, *J. Nonlinear Sci. Appl.*, 5(2), 74-83.
10. Karapinar E., Erhan I.M., (2011) Fixed point theorems for operators on partial metric spaces, *Appl. Math. Lett.*, 24(11), 1900-1904.
11. Long W., Shukla S., Radenović S., Radojenic S., (2013) Some coupled coincidence and common fixed point results for a hybrid pair of mappings in 0 - complete partial metric spaces, *Fixed Point Theory Appl.*, Article ID 145.

12. Matthews S.G., (1994) Partial metric topology, Proc. 8^{th} Summer Conference on General Topology and Applications, *Ann. New York Acad. Sci*., 728, 183-197.
13. Nashine H.K., Karapinar E., (2013) Fixed point results in orbitally complete partial metric spaces, *Bull. Malay. Math. Sci. Soc*., 36(4), 1185-1192.
14. Nashine H.K., Sintunavarat W., Kadelburg Z., Kumam P., Fixed point theorems in orbitally 0 - complete partial metric spaces via rational contractive conditions, Afr. Math. DOI 10.1007/s13370-014-0269-4.
15. Pathak H.K., (1986) Some nonunique fixed point theorems for a new class of mappings, *Ranchi Univ. Math. J.*, 17, 65-70.
16. Pathak H.K., (1988) On some nonunique fixed point theorems for the maps of Dhage type, *Pure Appl. Math. Sci*., 27, 65-70.
17. Popa V., (1997) Fixed point theorems for implicit contractive mappings, *Stud. Cercet. Ştiinţ., Ser. Mat., Univ. Bacău* 7, 129-133.
18. Popa V., (1999) Some fixed point theorems for compatible mappings satisfying an implicit relation, *Demonstr. Math*., 32(1), 159-163.
19. Romaguera S., (2010) A Kirk type characterization of completeness for partial metric spaces, *Fixed Point Theory Appl*., Article ID 493298.
20. Türkoğlu D., Özer O., Fisher B., (1999) Fixed point theorems for T - orbitally complete metric spaces, *Stud. Cercet. Ştiinţ., Ser. Mat., Univ. Bacău* 9, 211-218.
21. Vetro C., Vetro F., (2013) Common fixed points of mappings satisfying implicit relations in partial metric spaces, *J. Nonlinear Sci. Appl*., 6, 156-161.